**Chapter 4**

**Introduction to Probability**

**Learning Objectives**

1. Obtain an appreciation of the role probability information plays in the decision making process.

2. Understand probability as a numerical measure of the likelihood of occurrence.

3. Know the three methods commonly used for assigning probabilities and understand when they should be used.

4. Know how to use the laws that are available for computing the probabilities of events.

5. Understand how new information can be used to revise initial (prior) probability estimates using Bayes’ theorem.

**Solutions:**

1. Number of experimental Outcomes = (3)(2)(4) = 24

2. 

|  |  |  |  |
| --- | --- | --- | --- |
| ABC | ACE | BCD | BEF |
| ABD | ACF | BCE | CDE |
| ABE | ADE | BCF | CDF |
| ABF | ADF | BDE | CEF |
| ACD | AEF | BDF | DEF |

3. 

BDF BFD DBF DFB FBD FDB

4. a.



b. Let: H be head and T be tail

(H,H,H) (T,H,H)

(H,H,T) (T,H,T)

(H,T,H) (T,T,H)

(H,T,T) (T,T,T)

c. The outcomes are equally likely, so the probability of each outcome is 1/8.

5. *P*(Ei) = 1/5 for i = 1, 2, 3, 4, 5

*P*(Ei)  0 for i = 1, 2, 3, 4, 5

*P*(E1) + *P*(E2) + *P*(E3) + *P*(E4) + *P*(E5) = 1/5 + 1/5 + 1/5 + 1/5 + 1/5 = 1

The classical method was used.

6. *P*(E1) = .40, *P*(E2) = .26, *P*(E3) = .34

The relative frequency method was used.

7. No. Requirement (4.4) is not satisfied; the probabilities do not sum to 1. *P*(E1) + *P*(E2) + *P*(E3) + *P*(E4) = .10 + .15 + .40 + .20 = .85

8. a. There are four outcomes possible for this 2-step experiment; planning commission positive - council approves; planning commission positive - council disapproves; planning commission negative - council approves; planning commission negative - council disapproves.

b. Let p = positive, n = negative, a = approves, and d = disapproves



9. 

10. a. Using the table provided, 94% of students graduating from Morehouse College have debt.

*P***(**Debt) = .94

b. Five of the 8 institutions have over 60% of their graduates with debt.

*P*(over 60%) = 5/8 = .625

c. Two of the 8 institutions have graduates with debt who have an average debt more than $30,000.

*P*(more than $30,000) = 2/8 = .25

d. *P*(No debt) = 1 - *P***(**Debt) = 1 - .72 = .28

e. This is a weighted average calculation. 72% graduate with an average debt of $32,980 and 28% graduate with a debt of $0.

Average debt per graduate =

11. a. Total motorcyclists = 350 + 170 = 520

*P*(DOT-Compliant Helmet) =

b. Yes, the overall probability has been increasing from .48 five years ago, to .63 one year ago, and is now approximately .67. The probability that a motorcyclist wears a DOT-compliant helmet appears to be increasing.

c. Northeast: 

Midwest: 

South: 

West: 

The West region shows the highest probability (.8261) of DOT-compliant helmet use.

12. a. Use the counting rule for combinations:



One chance in 3,489,761

b. Very small: 1/3,478,761 = .000000287

c. Multiply the answer in part (a) by 42 to get the number of choices for the six numbers.

Number of Choices = (3,478,761)(42) = 146,107,962

Probability of Winning = 1/146,107,962 = .00000000684

13. Initially a probability of .20 would be assigned if selection is equally likely. Data does not appear to confirm the belief of equal consumer preference. For example using the relative frequency method we would assign a probability of 5/100 = .05 to the design 1 outcome, .15 to design 2, .30 to design 3, .40 to design 4, and .10 to design 5.

14. a. *P*(E2) = 1/4

b. *P*(any 2 outcomes) = 1/4 + 1/4 = 1/2

c. *P*(any 3 outcomes) = 1/4 + 1/4 + 1/4 = 3/4

15. a. *S* = {ace of clubs, ace of diamonds, ace of hearts, ace of spades}

b. *S* = {2 of clubs, 3 of clubs, . . . , 10 of clubs, J of clubs, Q of clubs, K of clubs, A of clubs}

c. There are 12; jack, queen, or king in each of the four suits.

d. For a: 4/52 = 1/13 = .08

For b: 13/52 = 1/4 = .25

For c: 12/52 = .23

16. a. (6)(6) = 36 sample points

b.



c. 6/36 = 1/6

d. 10/36 = 5/18

e. No. *P*(odd) = 18/36 = *P*(even) = 18/36 or 1/2 for both.

f. Classical. A probability of 1/36 is assigned to each experimental outcome.

17. a. (4,6), (4,7), (4,8)

b. .05 + .10 + .15 = .30

c. (2,8), (3,8), (4,8)

d. .05 + .05 + .15 = .25

e. .15

18. a. *P*(no meals) == .0222

b. *P*(at least four meals) = *P*(4) + *P*(5) + *P*(6) + *P*(7 or more)

== .8226

c. *P*(two or fewer meals) = *P*(2) + *P*(1) + *P*(0)

== .1048

19. a. A summary of the data provided in the exercise follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Response** | **United States** | **Great Britain** | **Total** |
| Yes | 187 | 197 | 384 |
| No | 334 | 411 | 745 |
| Unsure | 256 | 213 | 469 |
| **Total** | 777 | 821 | 1598 |

Probability = 334/777 = .4299

b. Probability = (411 + 213)/821 = .76

c. Probability = (334 + 411) /1598 = .4662

d. The probability that an investor in the United States thinks the government is adequately protecting investors is 187/777 =.2407; for investors in Great Britain the probability is 197/821 = .24. The two probabilities are almost identical; thus, there does not appear to be a difference between the perceptions of investors in these two countries with regard to the “Yes” response.

However, in part (a) we showed that the probability that an investor in the United States does not think the government is adequately protecting investors is.4299, or approximately .43; for investors in Great Britain the probability is 411/821 = .5006 or approximately .50. These results show a slightly higher probability that an investor in Great Britain will say that the government is not protecting investors adequately.

20. a. *P*(N) = 54/500 = .108

b. *P*(T) = 48/500 = .096

c. Total in 5 states = 54 + 52 + 48 + 33 + 30 = 217

*P*(B) = 217/500 = .434

Almost half the Fortune 500 companies are headquartered in these five states.

21. a. 

b. 

c. The Cause of Fatality that is least likely to occur is Fires and Explosions with a probability of



22. a. *P*(A) = .40, *P*(B) = .40, *P*(C) = .60

b. *P*(A ∪ B) = *P*(E1, E2, E3, E4) = .80. Yes *P*(A ∪ B) = *P*(A) + *P*(B).

c. Ac = {E3, E4, E5} Cc = {E1, E4} *P*(Ac) = .60 *P*(Cc) = .40

d. A ∪ Bc = {E1, E2, E5} *P*(A ∪ Bc) = .60

e. *P*(B∪ C) = *P*(E2, E3, E4, E5) = .80

23. a. *P*(A) = *P*(E1) + *P*(E4) + *P*(E6) = .05 + .25 + .10 = .40

*P*(B) = *P*(E2) + *P*(E4) + *P*(E7) = .20 + .25 + .05 = .50

*P*(C) = *P*(E2) + *P*(E3) + *P*(E5) + *P*(E7) = .20 + .20 + .15 + .05 = .60

b. A ∪ B = {E1, E2, E4, E6, E7}

*P*(A ∪ B) = *P*(E1) + *P*(E2) + *P*(E4) + *P*(E6) + *P*(E7)

= .05 + .20 + .25 + .10 + .05 = .65

c. A ∩ B = {E4} *P*(A ∩ B) = *P*(E4) = .25

d. Yes, they are mutually exclusive.

e. Bc = {E1, E3, E5, E6}; *P*(Bc) = *P*(E1) + *P*(E3) + *P*(E5) + *P*(E6)

= .05 + .20 + .15 + .10 = .50

24. Let E = experience exceeded expectations

M = experience met expectations

a. Percentage of respondents that said their experience exceeded expectations

= 100 - (4 + 26 + 65) = 5%

*P*(E) = .05

b. *P*(M ∪ E) = *P*(M) + *P*(E) = .65 + .05 = .70

25. Let M = male young adult living in his parents’ home

F = female young adult living in her parents’ home

a. *P*(M ∪ F) = *P*(M) + *P*(F) - *P*(M ∩ F)

= .56 + .42 - .24 = .74

b. 1 - *P*(M ∪ F) = 1 - .74 = .26

26. a. Let *D* = Domestic Equity Fund

*P*(*D*) = 16/25 = .64

b. Let *A* = 4- or 5-star rating

13 funds were rated 3-star of less; thus, 25 – 13 = 12 funds must be 4-star or 5-star.

*P*(*A*) = 12/25 = .48

c. 7 Domestic Equity funds were rated 4-star and 2 were rated 5-star. Thus, 9 funds were Domestic Equity funds and were rated 4-star or 5-star

*P*(*D* ∩ *A*) = 9/25 = .36

d. *P*(*D* ∪ *A*) = *P*(*D*) + *P*(*A*) - *P*(*D* ∩ *A*)

= .64 + .48 - .36 = .76

27. Let *A* = the event the ACC has a team in the championship game

*S* = the event the SEC has a team in the championship game

a. 

b. 

c. 

There is a low probability that teams from both the ACC and SEC will be in the championship game.

d. 

There is a high probability that a team from the ACC or SEC will be in the championship game.

e. *P*(Neither conference) = 

In this case, teams will most likely come from the Big Ten (6), Big East (4), Pac-10 (4), or Big 12 (3). Numbers shown are the number of times teams from these conferences have played in the national championship game over the previous 20 years.

28. Let: B = rented a car for business reasons

P = rented a car for personal reasons

a. *P*(B ∪ P) = *P*(B) + *P*(P) - *P*(B ∩ P)

= .54 + .458 - .30 = .698

b. *P*(Neither) = 1 - .698 = .302

29. a. *P*(*E*) =

*P*(*R*) =

*P*(*D*) =

b. Yes; *P*(*E* ∩ *D*) = 0

c. Probability = 

d. Let *F* denote the event that a student who applies for early admission is deferred and later admitted during the regular admission process.

Events *E* and *F* are mutually exclusive and the addition law applies.

*P*(*E* ∪ *F*) = *P*(*E*) + *P*(*F*)

*P*(*E*) = .3623 from part (a)

Of the 964 early applicants who were deferred, we expect 18%, or .18(964) students, to be admitted during the regular admission process. Thus, for the total of 2851 early admission applicants

*P*(*F*) =

*P*(*E* ∪ *F*) = *P*(*E*) + *P*(*F*) = .3623 + .0609 = .4232

Note: .18(964) = 173.52. Some students may round this to 174 students. If rounding is done, the answer becomes .4233. Either approach is acceptable.

30. a. 

b. 

c. No because *P*(A | B)  *P*(A)

31. a. *P*(A ∩ B) = 0

b. 

c. No. *P*(A | B)  *P*(A); ∴ the events, although mutually exclusive, are not independent.

d. Mutually exclusive events are dependent.

32. a. Row and column sums are shown.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Car | Light Truck | Total |
| U.S. | 87.4 | 193.1 | 280.5 |
| Non U.S. | 228.5 | 148.0 | 376.5 |
| Total | 315.9 | 341.1 | 657.0 |

A total of 657.0 thousand vehicles were sold.

Dividing each entry in the table by 657.0 provides the following joint probability table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Car | Light Truck | Total |
| U.S. | .1330 | .2939 | .4269 |
| Non U.S. | .3478 | .2253 | .5731 |
| Total | .4808 | .5192 | 1.0000 |

b. Let *U* = U. S. manufacturer

*N* = Non U.S. manufacturer

*C* = Car

*L* = Light Truck

Marginal probabilities: *P*(*U*) = .4269 *P*(*B*) = .5731

There is a higher probability that the vehicle was not manufactured by a U. S. auto maker. In terms of market share, non U.S. auto makers lead with a 57.3% share of vehicle sales.

Marginal probabilities: *P*(*C*) = .4808 *P*(*L*) = .5192

The light truck category which includes pickup, minivans, SUVs and crossover models has a slightly higher probability. But the types of vehicles are fairly even split.

c.  

If a vehicle was manufactured by one of the U.S. auto makers, there is a higher probability it will be in the light truck category.

d.  

If a vehicle was not manufactured by one of the U.S. auto makers, there is a higher probability it will be a car.

e. 

If a vehicle was a light truck, there is better than a 50-50 chance that it was manufactured by one of the U.S. auto makers.

f. There is a higher probability, and thus a larger market share for non U.S. auto makers. However, the U. S. auto makers are leaders in sales for the light truck category.

33. a.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Undergraduate Major | | |  |
|  |  | Business | Engineering | Other | Totals |
| Intended Enrollment Status | Full-Time | 0.270 | 0.151 | 0.192 | 0.613 |
| Part-Time | 0.115 | 0.123 | 0.149 | 0.387 |
|  | Totals | 0.385 | 0.274 | 0.341 | 1.000 |

b. *P*(Business) = 0.385, *P*(Engineering) = 0.274, and *P*(Other) = 0.341, so Business is the undergraduate major that produces the most potential MBA students.

c. 

d. 

e. Let *A* denote the event that student intends to attend classes full time in pursuit of an MBA degree, and let *B* denote the event that the student was an undergraduate Business major. Are events *A* and *B* independent? Justify your answer.

For independence, we must have that; from the joint probability table in part (a) of this problem, we have

*P(*A) = 0.613

*P(*B) = 0.385

So 

But



Because , the events are not independent.

34. a. Let O = flight arrives on time

Oc = flight arrives late

S = Southwest flight

U = US Airways flight

J = JetBlue flight

Given: *P*(O | S) = .834 *P*(O | U) = .751 *P*(O | J) = .701

*P*(S) = .40 *P*(U) = .35 *P*(J) = .25

*P*(O | S) =

 *P*(O ∩ S) = *P*(O | S)*P*(S) = (.834)(.4) = .3336

Similarly

*P*(O ∩ U) = *P*(O | U)*P*(U) = (.751)(.35) = .2629

*P*(O ∩ J) = *P*(O | J)*P*(J) = (.701)(.25) = .1753

Joint probability table

|  |  |  |  |
| --- | --- | --- | --- |
|  | On time | Late | Total |
| Southwest | .3336 | .0664 | .40 |
| US Airways | .2629 | .0871 | .35 |
| JetBlue | .1753 | .0747 | .25 |
| Total: | .7718 | .2282 | 1.00 |

b. Southwest Airlines; *P*(S) = .40

c. *P*(O) = *P*(S ∩ O) + *P*(U ∩ O) + *P*(J ∩ O) = .3336 + .2629 + .1753 = .7718

d. 

Similarly, 



Most likely airline is US Airways; least likely is Southwest

35. a. The total sample size is 200. Dividing each entry by 200 provides the following joint probability table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Pay Rent | | |  |
|  |  | Yes |  | No |  |
|  | Yes | .28 |  | .26 | .54 |
| Buy a Car |  |  |  |  |  |
|  | No | .07 |  | .39 | .46 |
|  |  | .35 |  | .65 |  |

b. Let *C* = the event of financial assistance to buy a car

*R* = the event of financial assistance to pay rent

Using the marginal probabilities, *P*(*C*) = .54 and *P*(*R*) = .35. Parents are more likely to provide their adult children with financial assistance to buy a car. The probability of financial assistance to buy a car is .54 and the probability of financial assistance to pay rent is .35.

c. 

d. 

e. Financial assistance to buy a car is not independent of financial assistance to pay rent, .

If there is financial assistance to buy a car, the probability of financial assistance to pay rent increases from .35 to .5185. However, if there is no financial assistance to buy a car, the probability of financial assistance to pay rent decreases from .35 to .1522.

f. 

36. a. We have that P(Make the Shot) = .93 for each foul shot, so the probability that Jamal Crawford will make two consecutive foul shots is that P(Make the Shot) P(Make the Shot) = (.93)(.93) = .8649.

b. There are three unique ways that Jamal Crawford can make at least one shot – he can make the first shot and miss the second shot, miss the first shot and make the second shot, or make both shots. Since the event “Miss the Shot” is the compliment of the event “Make the Shot,” P(Miss the Shot) = 1 – P(Make the Shot) = 1 – .93 = .07. Thus:

P(Make the Shot) P(Miss the Shot) = (.93)(.07) = .0651

P(Miss the Shot) P(Make the Shot) = (.07)(.93) = .0651

P(Make the Shot) P(Make the Shot) = (.93)(.93) = .8649

.9951

c. We can find this probability in two ways. We can calculate the probability directly:

P(Miss the Shot) P(Miss the Shot) = (.07)(.07) = .0049

Or we can recognize that the event “Miss both Shots” is the compliment of the event “Make at Least One of the Two Shots”, so

P(Miss the Shot) P(Miss the Shot) = 1 - .9951 = .0049

d. For the Portland Trail Blazers’ center, we have:

P(Make the Shot) = .58 for each foul shot, so the probability that the Portland Trail Blazers’ center will make two consecutive foul shots is P(Make the Shot) P(Make the Shot) = (.58)(.58) = .3364.

Again, there are three unique ways that the Portland Trail Blazers’ center can make at least one shot – he can make the first shot and miss the second shot, miss the first shot and make the second shot, or make both shots. Since the event “Miss the Shot” is the compliment of the event “Make the Shot,” P(Miss the Shot) = 1 – P(Make the Shot) = 1 – .58 = .42. Thus

P(Make the Shot) P(Miss the Shot) = (.58)(.42) = .2436

P(Miss the Shot) P(Make the Shot) = (.42)(.58) = .2436

P(Make the Shot) P(Make the Shot) = (.58)(.58) = .3364

.8236

We can again find the probability the Portland Trail Blazers’ center will miss both shots in two ways. We can calculate the probability directly:

P(Miss the Shot) P(Miss the Shot) = (.42)(.42) = .1764

Or we can recognize that the event “Miss both Shots” is the compliment of the event “Make at Least One of the Two Shots”, so

P(Miss the Shot) P(Miss the Shot) = 1 - .9951 = .1764

Intentionally fouling the Portland Trail Blazers’ center is a better strategy than intentionally fouling Jamal Crawford.

37. Let C = event consumer uses a plastic card

B = event consumer is 18 to 24 years old

Bc = event consumer is over 24 years old

Given information:









a. 

but is unknown. So first compute



Then



b. 

but andare unknown. However, they can be computed as follows.





Then



c. There is a higher probability that the younger consumer, age 18 to 24, will use plastic when making a purchase. The probability that the 18 to 24 year old consumer uses plastic is .5021 and the probability that the older than 24 year old consumer uses plastic is .3485. Note that there is greater than .50 probability that the 18 to 24 years old consumer will use plastic.

d. Companies such as Visa, Mastercard and Discovery want their cards in the hands of consumers who will have a high probability of using the card. So yes, these companies should get their cards in the hands of young consumers even before these consumers have established a credit history. The companies should place a low limit of the amount of credit charges until the young consumer has demonstrated the responsibility to handle higher credit limits.

38. a. The data table with row and column totals is shown below. Note that a total of 423,392 students took the state mathematics exam.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Met Proficiency Standards? | |  |
| Grade | Yes | No | Total |
| 3 | 47,401 | 23,975 | 71,376 |
| 4 | 35,020 | 34,740 | 69,760 |
| 5 | 36,062 | 33,540 | 69,602 |
| 6 | 36,361 | 32,929 | 69,290 |
| 7 | 40,945 | 29,768 | 70,713 |
| 8 | 40,720 | 31,931 | 72,651 |
| Total | 236,509 | 186,883 | 423,392 |

The joint probability table follows. The probabilities were computed by dividing each entry in the above table by the total number of students taking the state mathematics exam: 423,392.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Met Proficiency Standards | |  |
| Grade | Yes | No | Total |
| 3 | .11196 | .05663 | 0.16858 |
| 4 | .08271 | .08205 | 0.16476 |
| 5 | .08517 | .07922 | 0.16439 |
| 6 | .08588 | .07777 | 0.16365 |
| 7 | .09671 | .07031 | 0.16702 |
| 8 | .09618 | .07542 | 0.17159 |
| Total | .55861 | .44139 | 1.00000 |

For example, let *G*3 = event that a student is in the third grade and *S* = event that the student met the proficiency standards on the exam. The joint probability in the Grade 3 row and the Yes column is



b. The column marginal probabilities are .55861 and .44139. The marginal probability of .55861 is the probability that a randomly selected student met the proficiency standards on the exam and the marginal probability of .44139 is the probability that a randomly selected student did not meet the proficiency standards on the exam. There is a slightly better than 50-50 chance that a student met the proficiency standards on the exam.

The row marginal probabilities of .16858, .16476, and so on show the probability that a random selected student is in each grade. Because the number of students in each grade is approximately the same, the probabilities are very similar. Note that the highest probability of .17159 is the probability that a randomly selected student who took the exam is in the 8th grade.

c. 

Let *G*4 = event that a randomly student is in the fourth grade.



d. 



39. a. Yes, since *P*(A1 ∩ A2) = 0

b. *P*(A1 ∩ B) = *P*(A1)*P*(B | A1) = .40(.20) = .08

*P*(A2 ∩ B) = *P*(A2)*P*(B | A2) = .60(.05) = .03

c. *P*(B) = *P*(A1 ∩ B) + *P*(A2 ∩ B) = .08 + .03 = .11

d. 



40. a. *P*(B ∩ A1) = *P*(A1)*P*(B | A1) = (.20)(.50) = .10

*P*(B ∩ A2) = *P*(A2)*P*(B | A2) = (.50)(.40) = .20

*P*(B ∩ A3) = *P*(A3)*P*(B | A3) = (.30)(.30) = .09

b. 

c.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Events | *P*(A*i*) | *P*(B | A*i*) | *P*(A*i* ∩ B) | *P*(A*i* | B) |
| A1 | .20 | .50 | .10 | .26 |
| A2 | .50 | .40 | .20 | .51 |
| A3 | .30 | .30 | .09 | .23 |
|  | 1.00 |  | .39 | 1.00 |

41. S1 = successful, S2 = not successful and B = request received for additional information.

a. *P*(S1) = .50

b. *P*(B | S1) = .75

c. 

42. M = missed payment

D1 = customer defaults

D2 = customer does not default

*P*(D1) = .05 *P*(D2) = .95 *P*(M | D2) = .2 *P*(M | D1) = 1

a. 

b. Yes, the probability of default is greater than .20.

43. a. Let *M* = event that a putt is made

*P*(*M*) = 983,764/1,613,234 = .610

Note: The probability that a putt is missed is 

b. Let *A*= event that a PGA Tour player has a par putt







c. Let *B* = event that a PGA Tour player has a birdie putt







d. These probabilities indicate that there is a much higher probability of making a par put than a birdie putt. The authors of the article referenced in this exercise state that “even the best players … show evidence of loss aversion.” In other words, PGA Tour players tend to view missing a par putt as a loss and making a birdie putt as a gain. With regard to putting, their behavior indicates that they prefer avoiding losses to making gains. “On average this bias costs the best golfers over $1.2 million in tournament winnings per year.”

44. a. *P*(*A*1) = .095

*P*(*A*2) = .905

*P*(*W* | *A*1) = .60

*P*(*W* | *A*2) = .49

b.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Events | *P*(*Ai*) | *P*(*W*|*Ai*) | *P*(*Ai*∩*W*) | *P*(*Ai*|*W*) |
| *A*1 | 0.095 | 0.60 | 0.05700 | 0.1139 |
| *A*2 | 0.905 | 0.49 | 0.44345 | 0.8861 |
|  |  | *P*(*W*) = 0.50045 | | 1.0000 |

*P*(*A1*|*W*) = .1139

c.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Events | *P*(*Ai*) | *P*(*M*|*Ai*) | *P*(*Ai*∩*M*) | *P*(*Ai*|*M*) |
| *A1* | 0.095 | 0.40 | 0.03800 | 0.0761 |
| *A2* | 0.905 | 0.51 | 0.46155 | 0.9239 |
|  |  | *P*(*M*) = 0.49995 | | 1.0000 |

*P*(*A1*|*M*) = .0761

d*. P*(*W*) = .50045

*P*(*M*) = .49965

45. a. Let *A* = age 65 or older



b. Let *D* = takes drugs regularly

*P*() = 

= 

=  = .2485

46. a. Let *A* = a respondent owns a home

*P*(*A*) = 1249/2082 = .60

b. Let *B* = a respondent aged 18 to 34 owns a home

*P*(*B*) = 117/450 = .26

c. Let *AC* = a respondent does not own a home

*P*(*AC*) = 1 - *P*(*A*) = 1 - .60 = .40

d. Let *BC* = a respondent aged 18 to 34 does not own a home

*P*(*BC*) = 1 - *P*(*B*) = 1 - .26 = .74

47. a. (2)(2) = 4

b. Let S = successful

U = unsuccessful

Oil

Bonds

S

U

S

U

S

U

E1

E2

E3

E4

c. O = {E1, E2}

M = {E1, E3}

d. O ∪ M = {E1, E2, E3}

e. O ∩ M = {E1}

f. No; since O ∩ M has a sample point.

48. a. 0.5029

b. 0.5758

c. No, from part (a) we have P(*F*) = 0.5029 and from part (b) we have P(*A*|*F*) = 0.5758. Since P(*F*) ≠ P(*A*|*F*), events *A* and *F* are not independent.

49. Let I = treatment-caused injury

D = death from injury

N = injury caused by negligence

M = malpractice claim filed

$ = payment made in claim

We are given *P*(I) = 0.04, *P*(N | I) = 0.25, *P*(D | I) = 1/7, *P*(M | N) = 1/7.5 = 0.1333,

and *P*($ | M) = 0.50

a. *P*(N) = *P*(N | I) *P*(I) + *P*(N | Ic) *P*(Ic)

= (0.25)(0.04) + (0)(0.96) = 0.01

b. *P*(D) = *P*(D | I) *P*(I) + *P*(D | Ic) *P*(Ic)

= (1/7)(0.04) + (0)(0.96) = 0.006

c. *P*(M) = *P*(M | N) *P*(N) + *P*(M | Nc) *P*(Nc)

= (0.1333)(0.01) + (0)(0.99) = 0.001333

*P*($) = *P*($ | M) *P*(M) + *P*($ | Mc) *P*(Mc)

= (0.5)(0.001333) + (0)(0.9987) = 0.00067

50. a. Probability of the event = *P*(average) + *P*(above average) + *P*(excellent)

=  = .22 + .28 + .26 = .76

b. Probability of the event = *P*(poor) + *P*(below average)

= 

51. a.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Household Income ($1000) | | | | |  |
| Education Level | Under 25 | 25-49.9 | 50-74.9 | 75-99.9 | 100 or More | Total |
| Not H.S. Graduate | .0571 | .0469 | .0188 | .0073 | .0050 | .1351 |
| H.S. Graduate | .0667 | .0929 | .0682 | .0358 | .0362 | .2997 |
| Some College | .0381 | .0713 | .0634 | .0441 | .0553 | .2721 |
| Bachelor's Degree | .0120 | .0284 | .0386 | .0350 | .0729 | .1870 |
| Beyond Bach. Degree | .0039 | .0112 | .0173 | .0168 | .0568 | .1061 |
| Total | .1777 | .2508 | .2064 | .1390 | .2262 | 1.0000 |

b. This is a marginal probability.

*P*(Not H.S. graduate) = .1351

c. This is the sum of 2 marginal probabilities.

*P*(Bachelor's Degree ∪ Beyond Bachelor's Degree) = .1870 + .1061 = .2931

d. This is a conditional probability.



e. This is a marginal probability.

*P*(Under 25) = .1777

f. This is a conditional probability.



g. No. which is not equal to *P*(100 or More) = .2262. This is also shown

by comparing the probabilities in parts (e) and (f). Household income is not independent of education level. Individuals with a Bachelor’s Degree have a higher probability of having a higher household income.

52. a.



b. .2022

c. .2245 + .1283 + .1090 = .4618

d. .4005

53. a. *P*(24 to 26 | Yes) = .1482/.4005 = .3700

b. *P*(Yes | 36 and over) = .0253/.1090 = .2321

c. .1026 + .1482 + .1878 + .0917 + .0327 + .0253 = .5883

d. *P*(31 or more | No) = (.0956 + .0837)/.5995 = .2991

e. No, because the conditional probabilities do not all equal the marginal probabilities. For instance,

*P*(24 to 26 | Yes) = .3700  *P*(24 to 26) = .3360

54. a. .7766

b. 

c. 

d. The attitude about this practice is not independent of the age of the respondent. We can show this in several ways. One example is to use the result from part (b). We have



and



If the attitude about this practice were independent of the age of the respondent, we would expect these probabilities to be equal. Since these probabilities are not equal, the data suggests the attitude about this practice is not independent of the age of the respondent.

e. Respondents in the 50+ age category are far more likely to say this practice is NOT OKAY than are respondents in the 18-29 age category:





55. a. 

We have *P*(B | S) > *P*(B).

Yes, continue the ad since it increases the probability of a purchase.

b. Estimate the company’s market share at 20%. Continuing the advertisement should increase the market share since *P*(B | S) = .30.

c. 

The second ad has a bigger effect.

56. a. *P*(A) = 200/800 = .25

b. *P*(B) = 100/800 = .125

c. *P*(A ∩ B) = 10/800 = .0125

d. *P*(A | B) = *P*(A ∩ B)/*P*(B) = .0125/.125 = .10

e. No, *P*(A | B)  *P*(A) = .25

57. Let A = lost time accident in current year

B = lost time accident previous year

Given: *P*(B) = .06, *P*(A) = .05, *P*(A | B) = .15

a. *P*(A ∩ B) = *P*(A | B)*P*(B) = .15(.06) = .009

b. *P*(A ∪ B) = *P*(A) + *P*(B) - *P*(A ∩ B) = .06 + .05 - .009 = .101 or 10.1%

58. Let: B = blogger

Bc = non blogger

Y = young adult (18-29)

Yc = older adult

Given: *P*(B) = .08 *P*(Y | B) = .54 *P*(Y | Bc) = .24

*P*(Y | B) =

 *P*(Y ∩ B) = *P*(Y | B)*P*(B) = (.54)(.08) = .0432

*P*(Y | Bc) =

 *P*(Y ∩ Bc) = *P*(Y | Bc)*P*(Bc) = (.24)(.92) = .2208

|  |  |  |  |
| --- | --- | --- | --- |
|  | Young Adult | Older Adult | Total |
| Blogger | .0432 | .0368 | .08 |
| Non Blogger | .2208 | .6992 | .92 |
| Total: | .2640 | .7360 | 1.00 |

b. *P*(Y) = *P*(B ∩ Y) + *P*(Bc ∩ Y) = .0432 + .2208 = .2640

c. *P*(Y ∩ C) = .0432

d. *P*(B | Y) =

59. a. *P*(Oil) = .50 + .20 = .70

b. Let S = Soil test results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Events | *P*(Ai) | *P*(S | Ai) | *P*(Ai ∩ S) | *P*(Ai | S) |
| High Quality (A1) | .50 | .20 | .10 | .31 |
| Medium Quality (A2) | .20 | .80 | .16 | .50 |
| No Oil (A3) | .30 | .20 | .06 | .19 |
|  | 1.00 | *P*(S) = .32 | | 1.00 |

*P*(Oil) = .81 which is good; however, probabilities now favor medium quality rather than high quality oil.

60. a.





If a message includes the word *shipping!*, the probability the message is spam is high (0.7910), and so the message should be flagged as spam.

b.





A message that includes the word *today!* is more likely to be spam. This is because *P*(*today!*|spam) is larger than *P*(*here!*|spam). Because *today!* occurs more often in unwanted messages (spam), it is easier to distinguish spam from ham in messages that include *today!*.

c.





A message that includes the word *fingertips!* is more likely to be spam. This is because *P*(*fingertips!*|ham) is smaller than *P*(*available*|ham). Because *available* occurs more often in legitimate messages (ham), it is more difficult to distinguish spam from ham in messages that include *available*.

d. It is easier to distinguish spam from ham when a word occurs more often in unwanted messages (spam) and/or less often in legitimate messages (ham).